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1994 J. Phys.: Condens. Matter 6 1739

(http://iopscience.iop.org/0953-8984/6/9/015)

J. Phys.: Condens. Matter 6 (1994) 1739–1748. Printed in the UK

Thermal conductivity in the quantum Hall effect regime

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Received 13 July 1993, in final form 22 November 1993

Abstract. General expressions for both components of the thermal conductivity tensor of a two-dimensional electron gas in the presence of impurity disorder in a strong magnetic field are derived within the self-consistent Born approximation. Analytical and numerical results are obtained for asymptotic and intermediate cases respectively. It is found that the shape of the thermal conductivity dependence on the chemical potential is governed by the relation between temperature and impurity potential amplitude. Both thermal conductivity components as functions of chemical potential are shown to exhibit deviations from the Wiedemann–Franz law for finite temperatures near Landau level positions.

1. Introduction

Two-dimensional (2D) electron systems in a strong transverse magnetic field have attracted close attention since the discovery of the quantum Hall effect (for an extensive review see [1]). One of the most unclear problems in this field is the heat transport. The main difficulty that appears in heat transport studies originates from the fact that the temperature gradient is a non-mechanical disturbance, i.e. it cannot be represented directly as a perturbation of a microscopic Hamiltonian. Different ways to avoid this obstacle have been proposed. The first approach is to use the electron Lagrangian as a starting point for calculation of a heat current as an energy flow in Lagrange formalism [2, 3]. The second was originally proposed by Luttinger [4]. It incorporates the pseudo-gravitational field connected with ∇T through the Einstein relationship included in the Hamiltonian. Then the heat current can be calculated by the usual way with the diagrammatic technique based on Hamilton formalism, i.e. by means of Kubo formulae. Nevertheless, it can be shown in connection with quantum Hall effect problems that transverse (Hall) currents are screened by the surface ones in strong magnetic fields, which results in violation of the Kubo formulae for macroscopic (measurable) currents [5]. In this case non-diagonal components of the kinetic coefficient tensor can be expressed through the thermodynamic potential while for the diagonal ones the Kubo formulae remain valid.

With respect to the Luttinger approach the dependence of the thermopower tensor components on chemical potential, temperature and impurity concentration in the quantum Hall regime has been studied in detail [6–8]. The situation for the thermal conductivity (TC) tensor is obscure however. This problem was discussed by Oji [9, 10] who discovered that the dissipative (diagonal) component of the TC tensor has a characteristic 'two-peak' shape versus the chemical potential while the non-dissipative (non-diagonal) one exhibits plateaus similar to those for electrical conductivity. He also found that the Wiedemann–Franz law is violated for the diagonal component only while it holds for the Hall one. His approach however does not take into account correctly the effect of electron–impurity interaction on the kinetic coefficients.

0953-8984/94/091739+10$19.50

In the present paper analytic expressions for the TC tensor components in a 2D electron gas with impurity disorder in a quantized magnetic field are derived by means of the diagrammatic technique within the self-consistent Born approximation (SCBA). Both asymptotic expressions and numerical results for intermediate cases are obtained. It is shown that both diagonal and non-diagonal components of the TC tensor exhibit deviations from the Wiedemann–Franz law. In section 2 general expressions for both components of the heat current in the quantum Hall effect regime are derived. Section 3 and section 4 are devoted to the analysis of transverse and diagonal TC components respectively; results obtained are discussed in section 5.

2. Heat current in the quantum Hall effect regime

Below we follow the approach of Luttinger [4, 11, 12] to calculate the heat current as the response of a 2D electron system to the temperature gradient applied.

The phenomenological transport equations can be written as

$$\begin{aligned} I &= \hat{L}_{11}\left[E - \frac{T}{e}\nabla\left(\frac{\mu}{T}\right)\right] + \hat{L}_{12}\left[T\nabla\left(\frac{1}{T}\right) - \frac{1}{c^2}\nabla\psi\right] \\ Q &= \hat{L}_{21}\left[E - \frac{T}{e}\nabla\left(\frac{\mu}{T}\right)\right] + \hat{L}_{22}\left[T\nabla\left(\frac{1}{T}\right) - \frac{1}{c^2}\nabla\psi\right] \end{aligned} \tag{1}$$

where I and Q are macroscopic electric and heat currents respectively, $\nabla\psi$ is the pseudo-gravitational field, μ is the chemical potential, c is the velocity of light and $\hat{L}_{ij}$ are the transport coefficients. The thermal conductivity tensor can be expressed in these terms as $\hat{\kappa} = T^{-1}(\hat{L}_{22} - \hat{L}_{21}\hat{L}_{11}^{-1}\hat{L}_{12})$.). The phenomenological transport coefficients at $T\nabla(1/T)$ and $-\nabla\psi/c^2$ must be equal to each other (Einstein relationship). The origin of this fact is the absence of an energy current at equilibrium. Therefore, the problem is solved by introducing the pseudo-gravitational field that is a purely dynamic force (in contrast to the statistical one ∇T) and can be included in the Hamiltonian. So if starting from the expression for the heat current operator [13]

$$Q = -\frac{1}{2m}\left[\left(\nabla_r + \frac{ieA}{c}\right)\frac{\partial}{\partial t} + \left(\nabla_{r'} - \frac{ieA}{c}\right)\frac{\partial}{\partial t'}\right]_{r'\to r,\ t'\to t}\psi^+(r,t)\psi(r'\cdot t') - \frac{\mu}{e}I \tag{2}$$

$$I = \frac{ie}{2m}[\nabla_r - \nabla_{r'}]_{r'\to r}\psi^+(r',t)\psi(r\cdot t) - \frac{e^2}{mc}\psi^+\psi\mathbf{A} \tag{3}$$

(where e and m are the electron charge and mass respectively, ψ and ψ^+ are electron field operators in Heisenberg representation, A is the vector potential, the magnetic field B is directed along the z axis, and here and below we set $\hbar = 1$) we can obtain the Kubo formula [14, 2] (for the case of magnetic fields see [15]) by introducing the interaction representation and averaging the expression for heat current over the grand canonical ensemble:

$$\begin{aligned} Q_i(r,t) = -\frac{ie}{4m^2}\int \mathrm{d}r_1\,\mathrm{d}t_1 \Bigg[& \left(i\frac{\partial}{\partial t} - \mu\right)\frac{\partial}{\partial x_i} + \left(i\frac{\partial}{\partial t'} + \mu\right)\frac{\partial}{\partial x_i'} \\ & + \frac{eA_i(r)}{c}\left(i\frac{\partial}{\partial t} - i\frac{\partial}{\partial t'} - 2\mu\right)\Bigg]_{r'\to r,\ t'\to t}\Bigg[\left(i\frac{\partial}{\partial t_1} - \mu\right)\frac{\partial}{\partial x_{1k}} \\ & + \left(i\frac{\partial}{\partial t_1'} + \mu\right)\frac{\partial}{\partial x_{1k}'} + \frac{eA_k(r_1)}{c}\left(i\frac{\partial}{\partial t_1} - i\frac{\partial}{\partial t_1'} - 2\mu\right)\Bigg]_{r_1'\to r_1,\ t_1'\to t_1} \\ & \langle G(r_1', r, t_1' - t)G(r', r_1, t' - t_1)\rangle\Phi_k(r_1,t_1) + \frac{e\mu}{m}\Phi_i n_e. \end{aligned} \tag{4}$$

Here Φ is the potential of pseudo-gravitational field defined so that $\nabla T = -\partial\Phi/\partial t$, n_e is the density of electrons and G is the one-electron Green function in the coordinate representation. It is convenient to use the Keldysh diagrammatic technique [16] (see the review in [17]) to avoid the analytic continuation from Matsubara frequencies that is usual in the kinetic coefficient calculation. In this case the product of two G functions in equation (5) should be understood as a matrix product, and a vertex factor $\frac{1}{2}$ arises [17].

The Kubo formula in equation (4) defines the microscopic heat current as a response to ∇T. As was shown earlier [5] the macroscopic transverse currents are governed in strong magnetic fields not by the microscopic current (4) but by diamagnetic currents flowing on the sample surface. So equation (4) cannot be employed for the calculation of non-diagonal components of the TC tensor (as well as for the thermopower tensor). In the case under consideration the transverse component of the heat current can be expressed through the thermodynamic potential [18, 19, 11]:

$$Q_x = -\frac{cT}{eB}\left(\frac{\partial}{\partial T}\right)_\mu \int_{-\infty}^{\mu} \mathrm{d}\mu' S(\mu, T)\nabla_y T. \tag{5}$$

Here $S = -\partial\Omega/\partial T$ is the entropy per unit area. The above formulae are employed for the TC conductivity calculations.

3. Transverse component of the thermal conductivity

As was mentioned above to obtain the transverse TC component κ_{ik} (defined as $Q_i = -\kappa_{ik}\nabla_k T$) we have to calculate the thermodynamic potential Ω per unit area. The latter can be expressed in terms of the Green function [20]:

$$\Omega = \frac{1}{4\pi i}\int_{-\infty}^{\mu} \mathrm{d}\mu' \int_{-\infty}^{\infty} [G^{\mathrm{R}}(r, r, \omega) - G^{\mathrm{A}}(r, r, \omega)] \tanh\left(\frac{\omega - \mu'}{2T}\right) \mathrm{d}\omega. \tag{6}$$

Here G^{R} and G^{A} are retarded and advanced Green functions in the mixed coordinate-frequency representation respectively. These functions obey the Dyson equation that in the case of the electron–impurity interaction takes the form [20.21]:

$$G^{\mathrm{R,A}}(r, r', \omega) = G^{(0)\mathrm{R,A}}(r, r', \omega) + \sum_a \int \mathrm{d}r_1 G^{(0)\mathrm{R,A}}(r, r_1, \omega) u(r_1 - r_a) G^{\mathrm{R,A}}(r_1, r', \omega) \tag{7}$$

with $G^{(0)}$ being the Green function of free electrons, $u(r)$ and r_a are the impurity potential and position respectively; equation (7) should be averaged over impurity positions. A general way of describing the effect of impurities on electronic properties in strong magnetic fields is a very complicated problem that has not been solved exactly so far (see e.g. the discussion in [22] and [23]). However a certain progress can be achieved if one employs the SCBA [24, 6], which is valid for a weak short-ranged impurity potential. Let us assume the random impurity potential to be weakly disordered:

$$\langle u(r_1)u(r_2)\rangle = 2\pi l^2 \sigma^2 \delta(r_1 - r_2) \tag{8}$$

where σ is a characteristic energy to be interpreted as the impurity potential amplitude, $l = (c/eB)^{1/2}$ is the magnetic length and angular brackets denote the averaging over impurity

positions. If σ is much less than the chemical potential μ only diagrams in the lowest order of the impurity scattering should be taken into account. If the magnetic field in the Landau gauge is used ($A_y = Bx$) it is convenient to introduce 'momentum' (n, p_y, w) representation for the Green function (cf. [15]). One has

$$G^{\mathrm{R,A}}(x,x',y,y',\omega) = \sum_{n,n'} \int \mathrm{d}p_y\, \psi_n\left(x - \frac{cp_y}{eB}\right)\psi_{n'}\left(x' - \frac{cp_y}{eB}\right) \mathrm{e}^{\mathrm{i}p_y(y-y')} G^{\mathrm{R,A}}_{nn'}(p_y,\omega)$$
$$G^{(0)\mathrm{R,A}}_{nn'}(p_y,\omega) = \delta_{nn'}[\omega - \omega_n \pm \mathrm{i}\delta]^{-1} \tag{9}$$

with $\psi_n(x - cp_y/eB)$ and $\omega_n = (n + \frac{1}{2})\omega_c$ being the nth harmonic oscillator wave function and eigenvalue respectively, $\omega_c = eB/mc$ is the cyclotron frequency. In the SCBA the 'momentum' Green function is independent of momentum p_y and $G_{nn'} = G_n\delta_{nn'}$. In this representation equation (7) within the SCBA is rewritten as follows (see figure 1):

$$G_n(\omega) = G^{(0)}_n(\omega) + \sigma^2 G^{(0)}_n(\omega)\sum_{n'} G_{n'}(\omega)G_n(\omega) \tag{10}$$

which implies

$$G^{\mathrm{R,A}}_n(\omega) = [\omega - \omega_n - \Sigma^{\mathrm{R,A}}(\omega)]^{-1} \tag{11}$$

where $\Sigma^{\mathrm{R,A}}$ is the retarded/advanced self-energy, which satisfies the equation

$$\Sigma^{\mathrm{R,A}}(\omega) = \sigma^2 \sum_{k=0}^{\infty} [\omega - \omega_k - \Sigma^{\mathrm{R,A}}(\omega)]^{-1}. \tag{12}$$

Figure 1. The Dyson equation for the one-electron Green function within the SCBA: solid and double lines represent the Green function of the free electron and the dressed one respectively; a cross is an impurity potential while a dashed line denotes the averaging over impurity positions.

Finally, in the high-field approximation ($\omega_c \gg \sigma$) equation (12) is quadratic and can be easily solved [24, 6]:

$$\Sigma^{\mathrm{R,A}}(\omega) = \frac{\omega - \omega_L}{2} \mp \mathrm{i}\sigma\left[1 - \left(\frac{\omega - \omega_L}{2\sigma}\right)^2\right]^{\frac{1}{2}}. \tag{13}$$

Here ω_L is the energy of the Landau level nearest to the chemical potential. Using equations (6), (9), and (13) we obtain for the thermodynamic potential

$$\Omega = -\frac{eB}{c}\int_{-\infty}^{\mu} \mathrm{d}\mu' \int \frac{\mathrm{d}\omega}{2\pi^2} \tanh\left(\frac{\omega - \mu'}{2T}\right) \sum_n \frac{\mathrm{Im}\,\Sigma^{\mathrm{R}}}{(\omega - \omega_n - \mathrm{Re}\,\Sigma^{\mathrm{R}})^2 + (\mathrm{Im}\,\Sigma^{\mathrm{R}})^2}. \tag{14}$$

A term from the Landau level nearest to μ contributes to the sum in equation (14) only because terms corresponding to other Landau levels lead to corrections of order σ^2/ω_c^2 that

are to be omitted in the high-field approximation. An explicit form of self-energy (12) implies that the denominator of this term is exactly σ^2. After some algebra the expression for the non-diagonal TC can be reduced to a form

$$\kappa_{yx} = \int_{-\infty}^{\mu} d\mu' \sum_{n=0}^{\infty} \int_{-1}^{1} \frac{d\xi}{\pi^2} \frac{(\mu' - \omega_n - 2\sigma\xi)^2}{4T^2 \cosh^2[(\mu' - \omega_n - 2\sigma\xi)/2T]} (1-\xi^2)^{\frac{1}{2}} \quad (15)$$

where we have used the fact that $\kappa_{xy} = -\kappa_{yx}$. The above expression gives the general SCBA formula for the TC non-diagonal component that has to be investigated in different cases.

In the limiting case $\sigma \ll T \ll \omega_c$ (the pure electronic system) the first factor in equation (15) can be considered as independent of ξ. The expression for the TC is now reduced to

$$\kappa_{yx} = \int_{-\infty}^{\mu} \frac{d\mu'}{2\pi} \sum_{n=0}^{\infty} \frac{(\mu' - \omega_n)^2}{4T^2 \cosh^2[(\mu' - \omega_n)/2T]}. \quad (16)$$

This dependence has a step-like shape (see figure 2(*a*)). When $|\mu - \omega_L| \gg 4T$ the TC exhibits smooth plateaus with height $\pi Tn/6$ with n being the number of filled Landau levels. In the region of width approximately $4T$ from each half-filled Landau level ($\mu \simeq \omega_c(n + \frac{1}{2})$) the dependence shows steps each with one additional curve in the half height of the step. As is well known the Hall conductivity also shows steps as a function of chemical potential but these peculiarities are absent. So the Wiedemann–Franz (WF) law is violated in the limiting case $\sigma \ll T$. The value of the TC on the large plateau satisfies the WF law.

In the opposite limiting case of low temperatures or strong impurity potential ($T \ll \sigma \ll \omega_c$) the temperature is the lowest energy scale and the integral over $d\xi$ with the Fermi distribution function derivative in equation (15) can be treated as usual. This leads to

$$\kappa_{yx} = \frac{T}{6\sigma} \int_{\omega_n - 2\sigma}^{\mu} d\mu' \sum_{n=0}^{\infty} \left[1 - \left(\frac{\mu' - \omega_n}{2\sigma} \right)^2 \right]^{\frac{1}{2}} \quad (17)$$

where the fact is taken into account that only the region $\xi \simeq (\mu - \omega_n)/2\sigma$ contributes to the integral of equation (15).

Equation (17) also corresponds to a step function but one of rather different shape (figure 2(*a*)). The plateau height is the same that of the preceding case ($\pi Tn/6$), the step width is determined by the random potential and is now exactly 4σ (this is the range of energies where the imaginary part of self-energy (12) is non-zero). The shape of a step can be calculated exactly from equation (17):

$$\Delta\kappa_{xy} = \kappa_{xy}(\mu) - \kappa_{xy}(\omega_L) = \frac{T}{6} \left[\sin^{-1} \left(\frac{\mu - \omega_L}{2\sigma} \right) + \frac{\mu - \omega_L}{2\sigma} \left[1 - \left(\frac{\mu - \omega_L}{2\sigma} \right)^2 \right]^{\frac{1}{2}} \right]. \quad (18)$$

Results for intermediate cases cannot be obtained analytically. Numerical results for various σ/T according to equation (15) are shown in figure 2(*b*).

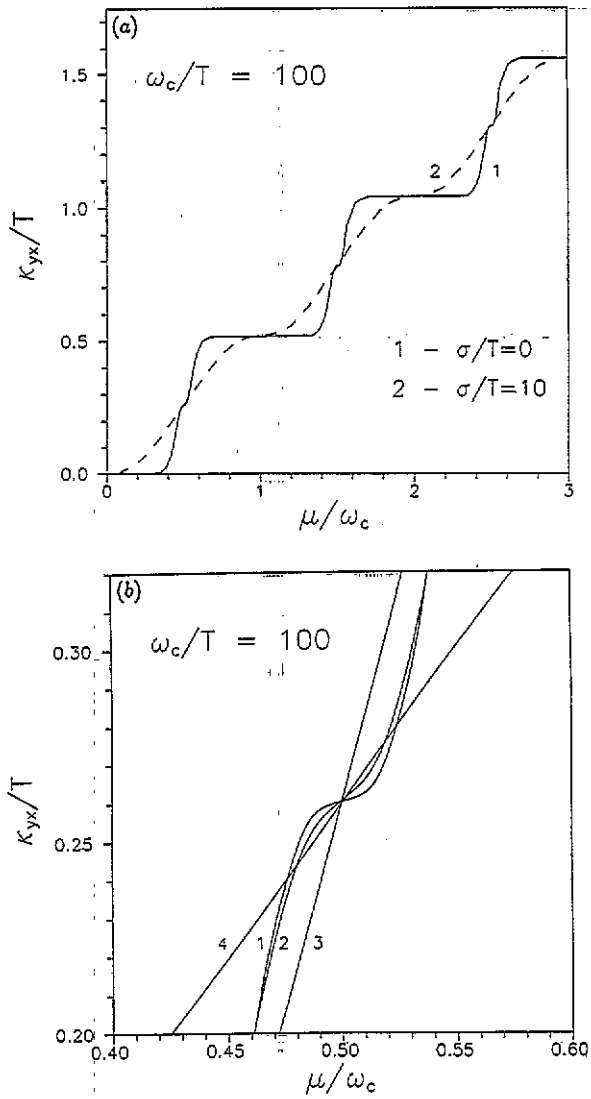

Figure 2. Results for the transverse TC component: κ_{xy}/T in units of $k_B^2/\hbar$ as a function of μ/ω_c; $\omega_c = 100T$ for various σ: (*a*) a general shape of the dependence; (*b*) the shape of the first step peculiarity for various σ/T: 0 (curve 1), 0.5 (curve 2), 1.5 (curve 3), 10 (curve 4).

4. Diagonal component of the thermal conductivity

It follows from equation (4) for the TC diagonal component that

$$\kappa_{xx} = \lim_{\omega_0 \to 0} \frac{1}{4m^2\omega_0 T} \mathrm{Re} \int_{-\infty}^{\infty} dx_1 \frac{dp}{2\pi} \frac{d\omega}{2\pi} (\omega - \mu)(\omega + \omega_0 - \mu) \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'} \right)_{x' \to x}$$
$$\times \left(\frac{\partial}{\partial x_1} - \frac{\partial}{\partial x_1'} \right)_{x_1' \to x_1} G(x_1', x, p, \omega) G(x', x_1, p, \omega + \omega_0) \tag{19}$$

The fact [24] that vertex corrections are absent in the SCBA is taken into account in equation (19). It is also convenient to use the 'momentum' representation $\{n, p, \omega\}$ as

was done in section 2. After the integrals over dx_1 and dp are calculated we obtain

$$\kappa_{xx} = -\frac{\omega_c^2}{16\pi^2 T^2}\int_{-\infty}^{\infty} d\omega \frac{(\omega-\mu)^2}{\cosh^2[(\omega-\mu)/2T]} \sum_{n=0}^{\infty} \frac{(n+1)(\operatorname{Im}\Sigma^A)^2}{|\omega-\omega_n-\Sigma^A|^2|\omega-\omega_{n+1}-\Sigma^A|^2}. \quad (20)$$

In the limiting case in question, $\sigma \ll \omega_c$, this expression can be simplified to the following form:

$$\kappa_{xx} = \frac{(n+1)}{8\pi^2}\frac{\sigma}{T^2}\int_{-1}^{1} d\xi(1-\xi^2)\frac{(\mu-\omega_n-2\sigma\xi)^2}{\cosh^2[(\omega_c-\mu)/2T]}. \quad (21)$$

This dependence has the form of peaks localized in the vicinity of points $\mu = \omega_n$. In the case when $T \ll \sigma \ll \omega_c$ the integral can be calculated as was done in section 3. The peak width is approximately T and its height (achieved at the point $\mu = \omega_n$) is $T(n+1)/12$ (figure 3(a)).

In the inverse case $\sigma \ll T \ll \omega_c$ the peak shape differs significantly from figure 3(a). Two maxima arise at points with $\mu \simeq \omega_n \pm 5T/2$ and the distance between the maxima does not depend on σ. On the contrary, the maximum value is governed by σ only and is about 0.03σ. At the point $\mu = \omega_n$ a minimum appears instead of the maximum in the preceding case; the value at this minimum is $2\sigma^3(n+1)/15\pi^2T^2$ and it is small in the case under consideration. The shape of the peak for different values of the ratio σ/T is shown in figure 3(b). One can see that the double-maximum shape tends to a single-maximum one as σ/T grows.

5. Discussion

Universal expressions for both components of thermal conductivity are obtained within the SCBA if the spin splitting of the Landau level is neglected. As is seen, the transverse TC component exhibits wide plateaus just like the transverse electrical conductivity (see [1]), and its plateau value exactly satisfies the WF law in all cases ($\kappa_{xy}/\sigma_{xy}T = \pi^2/3e^2$). This can be easily understood because the WF law is a consequence of the low-temperature expansion of integrals such as equation (15) with the derivative of the Fermi distribution function (see [25]). This expansion can be justified in the case under consideration when $|\mu - \omega_L| \gg \max(T, \sigma)$, which corresponds exactly to the plateau regions. So the plateau height is concerned with that in the transverse electrical conductivity. The situation is different around the steps. First, the shape of a step essentially depends on ratio σ/T: for $\sigma \gg T$ the step is sharp with width governed purely by σ; in this case the WF law holds for any values of chemical potential [9]. Finite temperature dramatically changes the situation however (figure 3). The step becomes wider and in the inverse limiting case $\sigma \ll T$ its width is governed by the temperature only. Moreover, an additional curve appears at the half width of the step also with width governed by the temperature. So the WF law breaks down at finite temperatures in the region around the step ($\mu \simeq \omega_c(n+\frac{1}{2})$).

The diagonal TC component shows a series of peaks as a function of chemical potential. The shape and height of the peaks strongly depend on the same parameter σ/T. In the limit $\sigma \gg T$ (rather high disorder or rather low temperatures), each peak has one maximum only corresponding to the half-filled Landau level ($\mu = \omega_c(n+\frac{1}{2})$); the peak width is governed by the impurity potential while its height is related to the temperature. The WF law holds in this case (as for the transverse TC component) in contrast with the conclusions of [10].

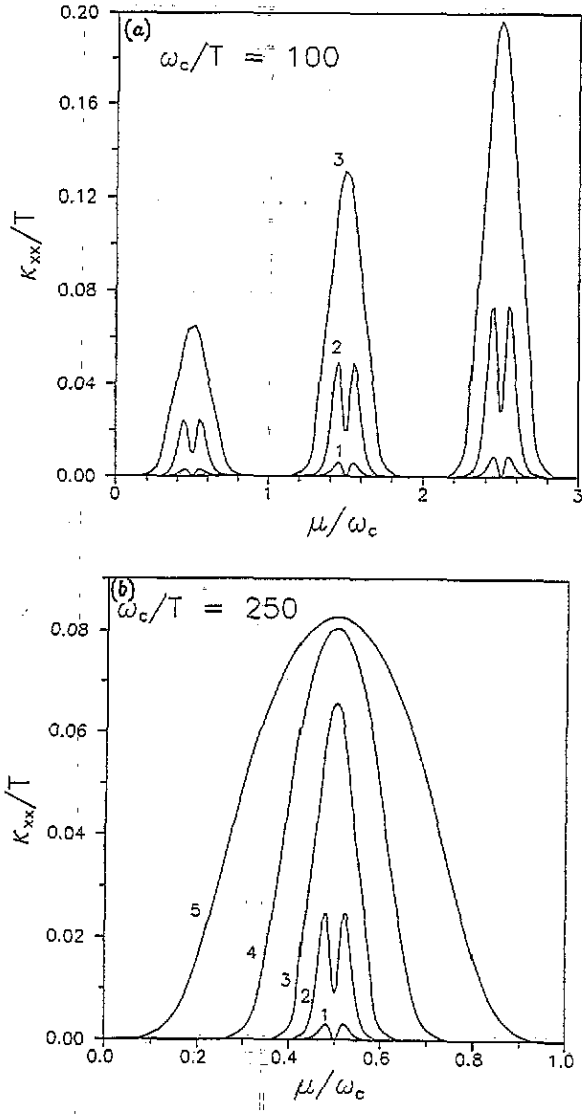

Figure 3. Results for the diagonal TC component: κ_{xx}/T in units of $k_B^2/\hbar$ as a function of μ/ω_c for various σ/T: 0.1 (curve 1), 1 (curve 2), 4 (curve 3), 10 (curve 4), 20 (curve 5). (*a*) A general shape of the dependence, $\omega_c = 100T$; (*b*) the shape of the first peak, $\sigma = 250T$.

When temperature is finite (σ/T drops) the peak broadens and the TC value at its maximum decreases. For $\sigma \ll T$ the shape of the peak is significantly different: the peak has two maxima and one minimum [10], the latter at $\mu = \omega_c(n + \frac{1}{2})$, and the distance between the maxima depends on the temperature only while its height is governed by the impurity potential. Because the diagonal electrical conductivity exhibits simple-maximum peaks for arbitrary σ/T [1, 10] one can conclude that the WF law breaks down for both TC components for finite temperatures in the vicinity of the Landau level positions.

The reasons for WF law violation can be clarified by consideration of TC components in terms of electric conductivity ($\hat{\sigma} = \hat{L}_{11}$) and thermopower ($\hat{S} = T^{-1}\hat{L}_{12}\hat{L}_{11}$). One can see that

$$\frac{\kappa_{xx}}{T\sigma_{xx}} = L_0 - S_{xx}^2 + S_{xy}^2 + 2\frac{\sigma_{xy}S_{xy}S_{xx}}{\sigma_{xx}}$$
$$\frac{\kappa_{xy}}{T\sigma_{xy}} = L_0 - S_{xx}^2 + S_{xy}^2 - 2\frac{\sigma_{xx}S_{xy}S_{xx}}{\sigma_{xy}} \tag{22}$$

where $L_0 = (\pi^2/3)(k_B/e)^2$ is the standard Lorentz number. So the source of deviation from the WF law is the fact that thermopower components are not small due to the rapid variation of electronic density of states. In particular, it is seen that the changing of the sign of S_{xy} midway between plateaus in σ_{xy} [6] is responsible for the appearance of a small extra plateau in κ_{xy}.

Let us briefly discuss the effects of spin splitting. As is known these effects must be taken into account in the expression for energy eigenvalues: $\omega_{n\sigma} = \omega_c(n + \frac{1}{2}) + \sigma g\mu_B B/2$ with σ being the spin projection. Each singularity (steps for the transverse TC and peaks for the diagonal one) splits now into two. In the case when the distance between the split Landau sublevels $g\mu_B B/2$ is much greater than $\max\{\sigma, T\}$ two separate singularities appear of the shape and size described above (with the height being half of the calculated value). In the case when this distance is approximately equal to $N\omega_c$, N being an integer, these singularities mix together and the dependence is more complicated than that described above.

We have also to compare our results with those obtained by Oji [9, 10]. Firstly it is worth noting that Oji's approach is of phenomenological rather than microscopic character and moreover only the case of weak disorder ($\sigma \ll T$) was considered in [9] and [10]. Our microscopical approach based on the Green function technique provides a correct way to take into account all interaction effects with arbitrary strength of disorder. In the limit of weak disorder our results appeared to be in qualitative agreement with [9] and [10], but more refined. As was mentioned above, strong disorder ($\sigma \gg T$) drastically alters the behaviour of TC components, leading to results never found by Oji.

In conclusion a few words on the possibilities of the experimental verification of effects described above are needed. First of all we note that measurements of electronic thermal conductivity in semiconducting structures are quite difficult because the lattice heat capacity dominates the electronic one and masks the effects of direct electron gas heating. Hence special methods to extract experimentally the electronic contribution to thermal conductivity are required, as was proposed for instance in [26]. In the usual experimental situation an impurity potential is quite strong and for fields $B \simeq 1$ T, σ is usually greater than the order of ω_c. In this case naturally $\sigma \gg T$ always and the only corresponding dependencies could be observed (the WF law is valid in this limiting case). To observe a different shape of dependences one requires rather strong magnetic fields (maybe of order 10 T) and low temperatures.

Acknowledgments

The authors are grateful to Professor Yu E Lozovik and Professor A S Siri for valuable discussions and to Professor V S Egorov for discussion of the experimental situation.

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